Renormalized Viscosity for High Acoustic Flux in Piezoelectric Semiconductors

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A renormalization procedure is described which gives the correction $\Delta\eta$ to the crystal viscosity η when high intensity acoustic flux propagates in a piezoelectric semiconductor. $\Delta\eta$ is attributed to the additional loss which a sound wave suffers from its production of forced higher harmonics via the electron gas. In the multimode case, the theory is based on the concept of "mode dressing". Numerical results are given for $\Delta\eta/\eta$ for an Akhieser type of small signal lattice loss and for conditions found in acoustoelectric domains. It was found that $\Delta\eta$ can exceed η appreciably when the frequency of the acoustic flux is one order of magnitude below the frequency of maximum linear electronic gain. The results are applied to the anomalous fast decay of low frequency phonons in high flux acoustoelectric domains.

1. Introduction

High acoustic flux intensities of the order kW/cm^2 or higher produced by acoustoelectric amplification in piezoelectric semiconductors [1] can alter the small signal characteristics of the crystal appreciably. The most significant effect is, of course, the electronic gain saturation through the intensity dependent renormalization of the electronic response function, i. e. "Nonlinear Acoustoelectric Sound Amplification". The complementary effect, namely the renormalization of the viscosity has not been extensively investigated.

In this paper, we discuss theoretically the renormalization $\Delta\eta$ of the viscosity η for the unperturbed crystal when high intensity acoustic flux propagates through a piezoelectric semiconductor. A formula for $\Delta\eta$ has been derived from the additional loss which a sound wave (or a whole group of waves in a small frequency band) suffers from the production of forced higher harmonics via the electron gas. These higher harmonics, though weak in amplitude [2], are subjected to increased nonelectronic losses due to the rapid increase in lattice loss with frequency. A similar loss mechanism has first been indicated in [3]. Early calculations of nonlinear lattice losses [4], refer only to frequencies equal to or greater than the frequency of maximum linear electronic gain [5]. In the following calculations it is demonstrated that this is an irrelevant frequency region for the renormalized viscosity.

The physical processes leading to the viscosity renormalization under discussion are shown schematically in Figure 1: Low frequency acoustic phonons are strongly coupled to the system of free conduction electrons via piezoelectric fields. The elec-
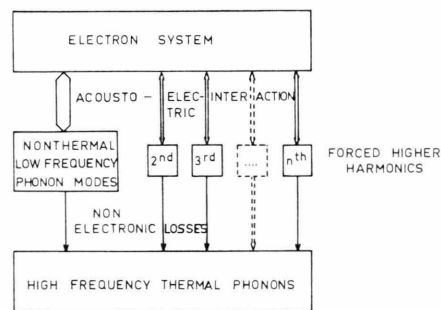

Fig. 1. Schematic diagram of the loss mechanism that requires renormalization of viscosity. The fundamental modes loose energy to forced higher harmonics via nonlinear coupling to the electron system. The harmonics are damped due to Akhieser type nonelectronic losses.

trons are bunched in the potential troughs of the alternating electric field (single mode case) [4, 6—9] or they are "dressed" by the combined effects of many such fields (multimode case) [10, 11]. The bunched electrons produce higher harmonics which through piezoelectric coupling are connected with forced higher harmonics of the fundamental phonon modes. The intrinsic crystal anharmonicity couples these low frequency phonons which in this case are highly nonthermal to thermal high frequency phonons. This results in dissipative processes. They are described by viscosities $\eta(n)$ $(n = 1, 2, 3, \ldots)$ by means of

which energy is extracted from the fundamental phonon modes either directly, i. e. $\eta(1)$, or via the forced higher harmonics, i. e.

$$\eta(2), \eta(3), \ldots, \eta(n), \ldots .$$

The latter processes lead to an effective renormalized viscosity for the fundamental phonon modes which is the object of the present study.

In the second section, the renormalization formalism is developed and used to obtain an expression for $\Delta\eta$. In the third section, this is evaluated numerically for a wide range of parameters and for an Akhieser type small signal viscosity η which is frequency independent [14, 15]. The main conclusion is that in a high flux acoustoelectric domain the renormalization $\Delta\eta$ cannot be neglected. Thus, these results yield new insight into the observed "anomallous" fast decay of low frequency acoustic phonons [16-18] in a high flux acoustoelectric domain (Section 4).

2. Theoretical Considerations

Consider a one-dimensional acoustic displacement field $u(x, t)$. Its Fourier decomposition under spatial quasihomogenity may be written as shown in Equation (1).

$$u(x, t) = \sum_{\varkappa} \tilde{u}(\varkappa, t) \cdot \exp\{-i\,\omega(\varkappa)\,t + i\,Q(\varkappa)\,x\} + \text{c. c.} \quad (1)$$

The index $\varkappa$ designates different modes essentially excited in the displacement field. It is assumed that the acoustic flux associated with the displacement field $u(x, t)$ is mainly concentrated around a mean value $\overline{Q}$ for the wave vectors Q while smaller portions of the flux are found around the wave vectors $n\,\overline{Q}$, $n = 2, 3, \ldots$. This situation is approximately realized either in high flux acoustoelectric domains [17, 19, 20], cf. also [21] or in the case of a single strong wave injected into the crystal [2].

In high acoustic flux, the "mode dressing" dominates the frequency mixing processes [11]. The concept of "mode dressing" is essentially [10] that the mere presence of high flux acts on any individual weak mode inside the (small) frequency band of the flux in the same way as the electron bunching acts on a single mode of the same *total intensity*. Hence, the differential equation [9] for one of the fundamental mode intensities $|\tilde{u}(\varkappa_1)|^2 \equiv u^2(\varkappa_1)$ is shown in Equation (2).

$$\partial_t u^2(\varkappa_1) = \omega(\varkappa_1)\,[K^2\,1/i\,\text{Im}\,1/\varepsilon_{nl} - K_\eta^2(\varkappa_1)]\,u^2(\varkappa_1). \quad (2)$$

Here, the time derivative is indicated by ∂_t, K^2 is the electromechanical coupling constant, and $K_\eta^2(\varkappa)$ defined in Eq. (3) measures the nonelectronic loss.

$$K_\eta^2(\varkappa) = \omega(\varkappa)\,\tilde{\eta}(\varkappa)/\tilde{c} . \quad (3)$$

$\tilde{\eta}(\varkappa)$ is the effective viscosity at the angular frequency $\omega(\varkappa)$ and $\tilde{c}$ is the unstiffened elastic constant.

Within the framework of the theories of nonlinear ultrasound amplification [9, 10] the nonlinear electronic response function ε_{nl} is given in the hydrodynamic limit by Equation (4).

$$\varepsilon_{nl} = 1 + (\varepsilon_{TF} - 1)\,\frac{1}{i\,y}\,\frac{J_{1-i\chi}(2\,i\,y)}{J_{-i\chi}(2\,i\,y)} . \quad (4)$$

Here, $\chi = (1 - v_d/v_s)\,\nu_D/\nu$ measures the activity of the electronic system (with v_d electronic drift velocity, v_s sound velocity, ν_D diffusion frequency, and ν mean frequency of the acoustic flux), ε_{TF} is the Thomas-Fermi dielectric constant for the wave vector $\overline{Q}$, and y is defined by Equation (5).

$$y^2 = \sum_{Q(\varkappa_1)\approx \overline{Q}} |\tilde{\varphi}(\varkappa_1)/k\,T|^2. \quad (5)$$

$\tilde{\varphi}(\varkappa_1)$ is the piezoelectric potential associated with the displacement mode $\tilde{u}(\varkappa_1)$.

By each $\tilde{u}(\varkappa_1)$ a whole spectrum of forced higher harmonics $\tilde{u}(\varkappa_n)$ $(n = 2, 3, \ldots;\ \omega(\varkappa_n) = n\,\omega(\varkappa_1))$ is generated by the nonlinear acoustoelectric interaction. They do not get much energy from the electrons because of phase mismatches due to the slightly nonlinear physical dispersion $\omega(Q)$. Therefore, the equations for the intensities $u^2(\varkappa_n)$ are as follows:

$$\partial_t u^2(\varkappa_n) = -\omega(\varkappa_n)\,K_\eta^2(\varkappa_n)\,u^2(\varkappa_n)\ ;\ n = 2, 3, \ldots . \quad (6)$$

It is clear that all equations and definitions also apply to the one mode case if one simply replaces $\varkappa_n$ by n.

Neither $K_\eta^2(\varkappa_1)$ nor $K_\eta^2(\varkappa_n)$ can be formed according to Eq. (3) using the small signal viscosity η. This becomes evident from Eq. (6) because these equations must admit growing higher harmonics connected with the growing fundamental wave. Thus $\tilde{\eta}(\varkappa_n)$ may be written as follows:

$$\tilde{\eta}(\varkappa_n) = \eta(\varkappa_n) + \Delta\eta(\varkappa_n);\ n = 1, 2, \ldots . \quad (7)$$

Here, $\eta(\varkappa_n)$ is the small signal viscosity and $\Delta\eta(\varkappa_n)$ the correction to it in intense acoustic flux. For $n = 1$ this renormalization is expected to be positive while for $n \geqq 2$ it is negative permitting gain conditions for the higher harmonics. One may therefore say that $\Delta\eta(\varkappa_n)$ accounts for the energy transfer between the fundamental wave and its associated higher harmonic spectrum. The sum of the energy flux rates $\partial_t I_{tr}(\varkappa_n)$ for this transfer process is zero and one has

$$\sum_{n \geq 1} \omega^4(\varkappa_n)\, \Delta\eta(\varkappa_n)\, u^2(\varkappa_n) = 0 . \qquad (8)$$

Here, it was used that I_{tr} is proportional to $\omega^2 (u^2)_{\text{transfer}}$. From Eqs. (6) and (8), it follows

$$\omega^4(\varkappa_1)\, u^2(\varkappa_1)\, \Delta\eta(\varkappa_1) = \sum_{n \geq 2} \omega^4(\varkappa_n)\, u^2(\varkappa_n)\, \eta(\varkappa_n)$$
$$+ \tilde{c} \sum_{n \geq 2} \omega^2(\varkappa_n)\, \partial_t u^2(\varkappa_n) . \qquad (9)$$

Introducing the acoustic fluxes

$$I(\varkappa_n) \propto n^2\, \omega^2(\varkappa_1)\, u^2(\varkappa_n)$$

this is simply written as

$$\Delta\eta(\varkappa_1) = \sum_{n \geq 2} n^2\, \eta(\varkappa_n)\, I(\varkappa_n)/I(\varkappa_1)$$
$$+ \tilde{c}/\omega^2(\varkappa_1) \sum_{n \geq 2} \partial_t I(\varkappa_n)/I(\varkappa_1) . \qquad (10)$$

The second term on the right hand side of Eq. (10) may be interpreted as a dynamical viscosity which results from the amplification of the higher harmonics. The first term represents that part of the nonelectronic loss experienced by the fundamental wave which is necessary to maintain a definite content of energy in the higher harmonics. This term is related to the sum of all acoustic energies in the higher harmonics which are dissipated to the thermal bath [22].

3. Evaluation of the Renormalized Viscosity

For a practical evaluation of Eq. (10) it is advisable to compare the magnitudes of both contributions to $\Delta\eta$. One expects $\partial_t I(\varkappa_n) \approx 2\, I(\varkappa_n)/\tau$ where τ is the amplitude rise time of the fundamental wave. Consequently $2\,\tilde{c}/(\omega^2(\varkappa_1)\,\tau)$ competes with $n^2\, \eta(\varkappa_n)$ in the two terms of Equation (10). One can also say that $2/\tau$ competes with $\omega(\varkappa_n)\, K_n^2(\varkappa_n)$ which is the small signal nonelectronic decay time of the n-th harmonic. Near a saturated state or when $\omega(\varkappa_1)\, K_n^2(\varkappa_1)$ competes with the electronic rise time of the fundamental wave, the first contribution in Eq. (10)

will dominate the dynamical viscosity.

For the following only this case will be considered:

$$\Delta\eta(\varkappa_1) = \sum_{n \geq 2} n^2\, \eta(\varkappa_n)\, I(\varkappa_n)/I(\varkappa_1) = \sum_{n \geq 2} n^2\, \eta(\varkappa_n)\, I_n/I_1 . \qquad (11)$$

Here, the *total* flux ratios are used rather than the harmonic intensity ratios associated with an individual mode. In terms of the single mode – multi mode correspondence developed in [10] both ratios are equal for many waves in a small frequency band [23]. Substituting the explicit expression for the ratio I_n/I_1 given in Eq. (25) of [9] one obtains Equation (12).

$$\Delta\eta(\varkappa_1) = \left| \frac{\varepsilon_{nl} - 1}{\varepsilon_{nl}} \right|^2 \sum_{n \geq 2} \eta(\varkappa_n) \left| \frac{J_{n - i\chi}(2\,i\,y)}{J_{1 - i\chi}(2\,i\,y)} \right|^2 . \qquad (12)$$

Equation (12) gives the difference $\Delta\eta$ between the effective viscosity experienced by an acoustic wave of the fundamental frequency and the small signal viscosity η in terms of a weighted sum over the small signal viscosities of the higher harmonics. The weighting factors depend on the frequency (through the frequency dependence of ε_{nl} and χ), on the *total* flux concentrated around this wave (represented by y^2), and on the activity χ of the medium.

4. Discussion

To evaluate $\Delta\eta$ given in Eq. (12) analytically seems impossible, since the quotients of Bessel functions are rather complicated especially for the range of parameters which are of practical interest. Equation (12) was evaluated numerically for an Akhieser type of lattice loss [14, 15]. In this case η is frequency independent. An Akhieser type of lattice loss under true small signal conditions has recently been verified for CdS in [24].

The numerical results summarized in Figs. 2 and 3 show that the effect of the renormalization cannot be neglected if the fundamental frequency ν is well below the frequency ν_m of maximum linear electronic gain [5] and the intensity of the acoustic flux corresponds to $2\,y \gg 1$. Both conditions are usually found in high flux acoustoelectric domains [25]. In the calculations, higher harmonics up to $n = 19$ were taken into account [26]. Note, that $|\chi_m| = |\chi|\,\nu/\nu_m$ was used as the activity parameter in Figs. 2 and 3 because $\Delta\eta$ is an even function of χ. Increasing deviations from the synchronous electric dc field reduce
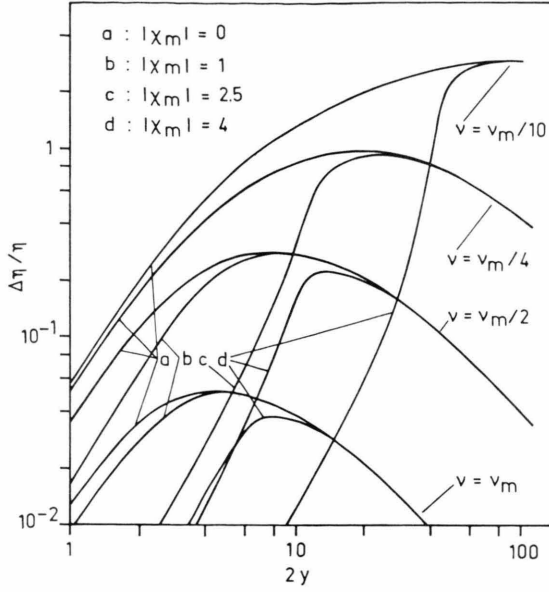
Fig. 2. Relative correction, $\Delta\eta/\eta$, to the small signal visco-sity, η, versus the bunching parameter, $2\,y$, for several values of the frequency ν and the activity parameter $\chi_m = (1-v_d/v_s)\,\nu_D/\nu_m$. ν_m is the frequency of maximum linear electronic gain.
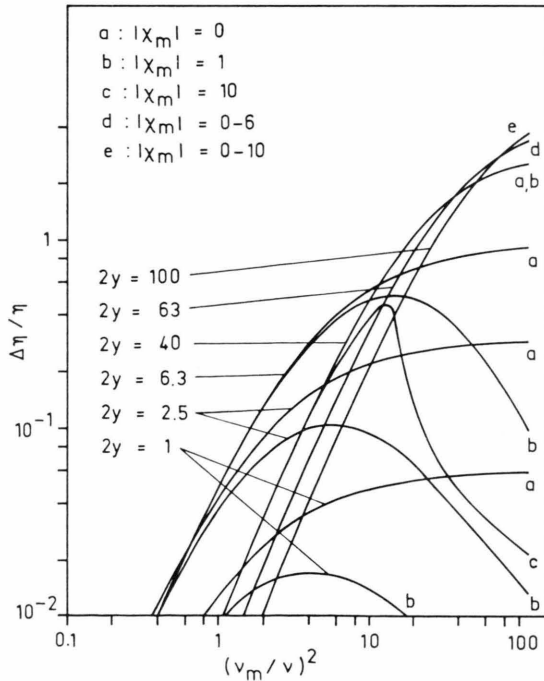

Fig. 3. Relative correction, $\Delta\eta/\eta$, to the small signal visco-sity, η, versus reduced frequency, ν_m/ν, for several values of the bunching parameter, $2\,y$, and the activity parameter $\chi_m = (1-v_d/v_s)\,\nu_D/\nu_m$. ν_m is the frequency of maximum linear electronic gain.

the higher harmonic generation and consequently the renormalization of the viscosity.

Special attention is drawn to the distinct differ-ence in the dependence of the renormalized elec-tronic gain, $\tilde{\alpha}$, and the renormalized nonelectronic viscosity, $\tilde{\eta}$, on $\delta \equiv v_d - v_s$. $\tilde{\alpha}$ is an odd function and $\tilde{\eta}$ is an even function of δ. Fortunately, the most pronounced effects on η occur for $\delta = 0$. $\Delta\alpha = \tilde{\alpha} - \alpha$ then cannot conceal the influence of $\Delta\eta = \tilde{\eta} - \eta$ where α is the small signal electronic gain [5].

The influence of the renormalization $\Delta\eta$ cannot be neglected if the following three conditions are met, i. e. high flux intensities, frequencies ν well below ν_m, and local dc electric field near the syn-chronous field. All conditions are fulfilled in a high flux acoustoelectric domain when the external field is suddenly switched off. Then the reversed acousto-electric effect produces an after current close to the synchronous current.

It was observed earlier [27] that the acoustic flux decays nonexponentially during the initial stages of the decay. This effect was attributed to second harmonic generation [3]. Later, the subject was further explored along these lines in [16]. In [16], it was shown that the anomalous fast nonlinear decay of acoustic flux in CdS refers to low frequency phonons in the region of some hundred MHz. Some quantitative re-sults have been given on which our first example in Table 1 is based. Qualitative descriptions of the same observation in GaAs have been given in [17] and [18]. Our second example refers to data extracted from [17].

Direct use of Figs. 2 and 3 require the knowledge of $|\chi_m|$, ν, and $2\,y$, the latter being the more in-volved quantity. The total flux density I_1 of the fun-damental phonon modes is related to y as shown in Equation (13) [9].

$$I_1 = A\,\nu^2\,|\,\varepsilon_{n1}\,|^2\,y^2. \tag{13}$$

Here, $A = (2\,\pi/v_s)\,(k\,T)^2/(e^2\,K^2/\varepsilon_0)$. For shear wave acoustic flux at room temperature, the parameter A takes the following values

$$A\,[\mathrm{W\,s^2\,cm^{-2}}] = \begin{cases} 3.4\times10^{-18} & \text{for ZnO} \\ 7.1\times10^{-18} & \text{for CdS} \\ 2.8\times10^{-17} & \text{for GaAs}. \end{cases} \tag{14}$$

A reasonable estimate for $|\,\varepsilon_{n1}\,|$ in an acoustoelectric domain with extreme high flux at frequencies well below ν_m is given by $|\,\varepsilon_{n1}\,| \approx 2$. The frequency ν then is approximately given by $\nu \approx \nu_m(y)^{-1/2}$. A more detailed treatment of this approximation is given in the Appendix.

Within the framework of the present theory, ν must be taken as the frequency around which the acoustic flux is mainly concentrated.

In [16], this frequency seems to be 700 MHz indicating an intermediate domain growth stage. The observed frequency ν_0, however, was 400 MHz [28]. Using the data [16] for CdS given in Table 1 one calculates $\Delta\eta/\eta = 0.8$. On the basis of a model involving only second harmonic generation the authors of [16] determine a value of about $\Delta\eta/\eta = 0.5$.

Note, that in the present theory the amount of higher harmonic generation in high acoustic flux neither obeys simple power laws (i. e. $I_n \propto I_1^{n\,3}$) nor is the assumption [16] correct that second harmonic generation plays a predominant role in the observed anomalous fast decay of low frequency acoustic phonons. The true situation is more complex, i. e. concerning the loss mechanism under discussion many higher harmonics are of importance. The generation of higher harmonics and consequently the values of $\Delta\eta/\eta$ exhibit a strong saturation tendency [9, 29]. This nonlinear saturation is clearly demonstrated in Figures 2 and 3. Consider, e. g. Fig. 3 and the frequency $\nu = \nu_m/5$. The data in Fig. 3 imply that in the range $2\,y = 6.3$ to $2\,y = 100$ the values of $\Delta\eta/\eta$ are approximately constant. Finally, one must stress the fact, that in the concept of "dressing", the generation of higher harmonics is *not* determined by the intensity of a single mode but by the combined effects of all excited modes. Therefore, only the *total* acoustic flux, I_1, enters our equations.

Thus, the above discussion overcomes the argument in [18] that the observed anomalous fast decay of low frequency phonons cannot be due to higher harmonic generation because the effect does not show the strong intensity dependence expected for (small signal) harmonic generation.

The data for GaAs contained in Table 1 were taken from [17]. In this work, no numerical values are given for the fast nonlinear decay of frequencies observed well below ν_m. However, using the values for the intensity and the mean frequency in a domain obtained from Figs. 11 and 13 of [17] and the present theory, one calculates $\Delta\eta/\eta = 1.6$. This result again shows the importance of the renormalization of the viscosity in high acoustic flux for a realistic experimental situation.

Acknowledgement

Appendix

Evaluation of $|\varepsilon_{nl}|$ is intimately related to the question of how the displacement amplitudes $|\tilde{u}(\varkappa_1)|$ and the associated potential amplitudes $|\tilde{\varphi}(\varkappa_1)|$ are related.

The statement in [6], i. e. that in the single mode case this relation is approximately linear, is debatable. The general relation between these quantities (c. f. [9]) is

$$|\varepsilon_{nl}|\,|\tilde{\varphi}(\varkappa_1)| = e\,(4\,\pi\,p/\varepsilon_0)\,|\tilde{u}(\varkappa_1)| \qquad (A\,1)$$

where p is the piezoelectric constant and ε_0 is the static dielectric constant. On the basis of the expression for ε_{nl} given in Eq. (4) one can state that $|\varepsilon_{nl}|$ varies between ε_{TF} and 1. The maximum value ε_{TF} is obtained for $\chi = 0$ and $2\,y = 0$. For fixed $2\,y$ and increasing $|\chi|$, $|\varepsilon_{nl}|$ decreases monotonically and approaches 1. For fixed $|\chi|$, $|\varepsilon_{nl}|$ equals 1 in the limit $2\,y \gg 1; |\chi|$. However, maxima of $|\varepsilon_{nl}|$ occur for $|\chi| \approx 2\,y$. For $\varepsilon_{TF} \gg 1$, which corresponds to

Table 1. Survey of data for two specific experiments on acoustoelectric domains.

Crystal		CdS	Ref.	GaAs	Ref.
Temperature T	(°K)	300	[16]	300	[17]
Driftmobility μ	(cm² V⁻¹ s⁻¹)	300	[16]	7000	[17]
Frequency ν_m of max. lin. electr. gain	(Hz)	2.6×10^9	[16]	4.5×10^9	[17]
Diffusion frequency ν_D	(Hz)	6×10^8		9×10^7	
Mean frequency ν of acoustic flux	(Hz)	7×10^8		7×10^8	[17]
Investigated frequency ν_0	(Hz)	4×10^8	[16]	6×10^8	[17]
Acoustic flux density I_1	(Wcm⁻²)	3×10^3	[16]	1×10^5	[17]
Bunching parameter $2\,y$		30		80	
Activity parameter $|\chi_m|$		0		<1	
Frequency ratio $(\nu_m/\nu)^2$		14		40	
Relative viscosity correction $\Delta\eta/\eta$		0.8		1.6	
Experimental estimate for $\Delta\eta/\eta$		0.5	[16]	—	

$\nu < \nu_{\mathrm{m}}$, $|\varepsilon_{\mathrm{nl}}|$ varies rapidly as a function of $|\chi|$ and $2y$. Therefore, a linear relationship between $|\tilde{u}|$ and $|\tilde{\varphi}|$ does not hold in the single mode case. In the multimode case, each individual mode is weak and the relation between $|\tilde{u}|$ and $|\tilde{\varphi}|$ for each mode is approximately linear. However, the relation between the two is given by Eq. (A 1) and contains $|\varepsilon_{\mathrm{nl}}|$ as the manifestation of the "dressing" effect due to the presence of many other modes.

There is a convenient result in the limit $2y > |\chi|$ and $2y \gg 1$. This condition is characteristic for a truly high flux acoustoelectric domain. In this limit,

and with the additional assumption, $\nu = \tilde{\nu} = \nu_{\mathrm{m}}(y)^{-1/2}$ (see Ref. [21]), where $\tilde{\nu}$ is the appropriate frequency of maximum nonlinear electronic gain, one obtains $|\varepsilon_{\mathrm{nl}}| \to 2$. This can easily be seen from the explicit expression

$$\varepsilon_{\mathrm{nl}} = 1 + (\varepsilon_{\mathrm{TF}} - 1)\,[1/y + i\,\chi/(2\,y^2)] \qquad (\mathrm{A}\,2)$$

for $|\varepsilon_{\mathrm{nl}}|$ valid under the above conditions if $\varepsilon_{\mathrm{TF}} - 1 \equiv (\nu_{\mathrm{m}}/\nu)^2$ is used. Note, that $|\varepsilon_{\mathrm{nl}}| = 2$ is the analogous relation in the strongly nonlinear regime to the small signal condition $\varepsilon_{\mathrm{TF}} = |\varepsilon_{\mathrm{TF}}| = 2$ for maximum linear electronic gain.

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